Dilepton production from non-equilibrium hot hadronic matter

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It is investigated under which conditions an adiabatic adaption of the dynamic and spectral information of vector mesons to the changing medium in heavy ion collisions, as assumed in schematic model calculations and microscopic transport simulations, is a valid assumption. Therefore time dependent medium modifications of low mass vector mesons are studied within a nonequilibrium quantum field theoretical description. Timescales for the adaption of the spectral properties are given and non-equilibrium dilepton yields are calculated, leading to the result that memory effects are not negligible for most scenarios.

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I. INTRODUCTION AND MOTIVATION

High energy heavy ion reactions allow for studying strongly interacting matter under extreme conditions, i.e., high densities and temperatures. Photons and dileptons do not undergo strong interactions and thus may carry undistorted information on the early hot and dense phases of the fireball, because the production rates increase rapidly with temperature. Dilepton spectra are expected to play a central role in inferring the restoration of the spontaneous breaking of chiral symmetry from heavy ion reactions. In the low mass region they couple directly to the light vector mesons and reflect their mass distribution. They are thus considered the prime observable in studying mass (de-)generation related to the restoration of spontaneous chiral symmetry breaking.

In this work we investigate dilepton production from the hot hadronic medium and we will concentrate on the low mass region, particularly the medium modifications of the ρ-meson. The medium and hence also possibly the properties of the regarded mesons undergo substantial changes over time. Such scenarios have been described within transport calculations using some quantum mechanically inspired off-shell propagation [1, 2]. It emerges the important question, whether a quasi instantaneous adaption of the dynamic and spectral information to the changing medium, as assumed in more schematic fireball model calculations [3] and microscopic transport simulations, is a suitable assumption or whether the vector meson's spectral information reacts to changes with a certain "quantum mechanical" retardation. The transport description of off-shell excitations is an open field of research and necessary in order to understand the transport dynamics of resonances. We employ a nonequilibrium quantum field theoretical description based on the formalism established by Schwinger and Keldysh [4, 5]. We give a formula for the dynamic dilepton production rate and simulate modifications of the light vector mesons due to the dynamically changing medium in heavy ion collisions by parameterizing a certain time dependence of the ρ-meson self energies. We are able to analyze the mesons' dynamic spectral properties as well as the resulting dilepton rate and the yield from an evolving fireball and compare to the quantities computed assuming adiabaticity.

II. THE NONEQUILIBRIUM PRODUCTION RATE

We utilize the Schwinger-Keldysh formalism in order to derive the dynamic non-equilibrium rate of produced electron-positron pairs, coming from the decay of light vector mesons via virtual photons in a spatially homogeneous system (details in [6]).

Projecting on the particle number in the electron propagator $G^<$ and using the equations of motion for $G^<$, the so called Kadanoff-Baym equations, we find the production rate of electrons for a homogeneous, yet time dependent system, to read

$$\partial_\tau N(\mathbf{p},\tau) = 2\ \mathrm{Im}\left[\mathrm{Tr}\left\{\frac{\not p + m}{2E_{\mathbf{p}}}\int_{t_0}^{\tau} d\bar{t}\left(\Sigma^<(\mathbf{p},\tau,\bar{t})\right)e^{iE_{\mathbf{p}}(\tau-\bar{t})}\right\}\right], \quad (1)$$

with the electron self energy Σ and $p_0 = E_{\mathbf{p}}$. The free electron propagator can be used because due the electrons' long mean free path the electrons are not expected to interact with the medium after being produced.

The medium effects enter via the dressing of the virtual photon propagator in the electron self energy (see Fig. 1).

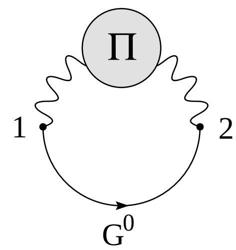

FIG. 1: $\Sigma(1,2)$ in coordinate space

D_γ is the virtual photon propagator and Π is its self energy. We have

$$i\Sigma^<(\mathbf{p},t_1,t_2) = -e^2\gamma_\mu \left(\int \frac{d^3k}{(2\pi)^3} D_\gamma^{<,\mu\nu}(\mathbf{k},t_1,t_2) G_0^<(\mathbf{p}-\mathbf{k},t_1,t_2) \right) \gamma_\nu, \tag{2}$$

with $\mathbf{k}$ the momentum of the virtual photon. On inserting this self energy and defining $p^+ = k - p$ as the four-momentum of the positron and $p^- = p$ as that of the electron, equation (1) becomes

$$E_+E_- \frac{dR}{d^3p^+d^3p^-}(\tau) = \frac{2e^2}{(2\pi)^6}\left[p_\mu^+p_\nu^- + p_\nu^+p_\mu^- - g_{\mu\nu}(p^+p^- + m^2)\right] \mathrm{Re}\left[\int_{t_0}^{\tau} d\bar{t} iD_\gamma^{<,\mu\nu}(\mathbf{k},\tau,\bar{t}) e^{i(E_++E_-)(\tau-\bar{t})}\right], \tag{3}$$

with $E_+ = E_\mathbf{p}$ and $E_- = E_{\mathbf{k}-\mathbf{p}}$.

Applying the equilibrium properties of $D^<$, it can be shown that equation (3) is the generalization of the well known thermal production rate for lepton pairs in the stationary case [7]:

$$E_+E_- \frac{dR}{d^3p^+d^3p^-}(\tau) = -\frac{2e^2}{(2\pi)^6}\left[p_\mu^+p_\nu^- + p_\nu^+p_\mu^- - g_{\mu\nu}(p^+p^- + m^2)\right] \frac{1}{M^4}\frac{1}{e^{\beta E}-1} \mathrm{Im}\Pi_\gamma^{\mathrm{ret},\mu\nu}(k,\tau) \tag{4}$$

In the following, we will consider the mode $\mathbf{k} = 0$ exclusively, i.e., the virtual photon resting with respect to the medium. After projecting on the virtual photon momentum and taking the electron mass to zero, we get

$$\frac{dN}{d^4xd^4k}(\tau,\mathbf{k}=0,E) = \frac{2e^2}{(2\pi)^6}\frac{2}{3}\pi(k_\mu k_\nu - k^2 g_{\mu\nu})\mathrm{Re}\left[\int_{t_0}^{\tau} d\bar{t} iD_\gamma^{<,\mu\nu}(\mathbf{k}=0,\tau,\bar{t}) e^{iE(\tau-\bar{t})}\right]. \tag{5}$$

The dynamic information is inherent in the memory integral on the right that runs over all virtual photon occupation numbers $D_\gamma^<$ from the initial time to the present. This way the full nonequilibrium electron production rate at the present time τ is determined. We introduce the dynamic medium dependence by dressing the virtual photon propagator with the medium dependent ρ- or ω-meson. This dressing enters with the self energy $\Pi^<$ via the relation

$$D^{\gtrless}(1,1') = \int_{t_0}^{\infty} d2 \int_{t_0}^{\infty} d3 D^+(1,2)\Pi^{\gtrless}(2,3)D^-(3,1') + \text{surface term}, \tag{6}$$

that follows from the Kadanoff-Baym equations. Using projectors on the different polarizations and the equality of each polarization ($\mathbf{k} = 0$), we find

$$\frac{dN}{d^4xd^4k}(\tau,E,\mathbf{k}=0) = \frac{2}{3}\frac{e^2}{(2\pi)^5}(3E^2)\mathrm{Re}\left[\int_{t_0}^{\tau} d\bar{t} iD_{\gamma,T}^<(\mathbf{k}=0,\tau,\bar{t}) e^{iE(\tau-\bar{t})}\right]. \tag{7}$$

For dilepton production $\Pi^{\mathrm{ret}} \propto e^2$ and E is the invariant mass of the virtual photon. For the cases we are interested in, $|\Pi^{\mathrm{ret}}| \ll E$ and we can approximate $D_T^< = D_0^{\mathrm{ret}} \otimes \Pi_T^< \otimes D_0^{\mathrm{av}}$. Diverging contributions at early times (low frequencies), due to the undamped photon propagators, cause numerical problems. We introduce an additional cutoff Λ for these propagators: $D_0^{\mathrm{ret}}(\tau - t_1) = (\tau - t_1) \to (\tau - t_1)e^{-\Lambda(\tau - t_1)}$ and analogously for $D_0^{\mathrm{av}}(t_2 - \bar{t})$. The exponential factors lead to a reduction of the rate, which we will overcome by renormalizing the final result. This does not affect the timescales we are interested in, and comparison of the dynamically computed rate for a stationary situation (constant self energy) with the analytic, thermal rate shows perfect agreement.

Vector meson dominance (VMD) allows for the calculation of the photon polarization tensor $\Pi^<_T$, using the identity between the electromagnetic current and the canonical interpolating fields of the vector mesons, which leads to $\Pi^<_{\alpha\beta} = \frac{e^2}{g_\rho^2} m_\rho^4 D^<_{\rho,\alpha\beta}$. Once more, we apply the generalized fluctuation dissipation relation (6) to calculate

$$D^<_{\rho,T} = D^{\mathrm{ret}}_{\rho,T} \otimes \Sigma^<_{\rho,T} \otimes D^{\mathrm{av}}_{\rho,T}, \tag{8}$$

with the ρ-meson self energy $\Sigma^<_\rho$. The transverse parts of the retarded and advanced propagators $D^{\mathrm{ret}}_{\rho,T}(\mathbf{k}, t_1, t_3) = D^{\mathrm{av}}_{\rho,T}(\mathbf{k}, t_3, t_1)$ of the vector meson in a spatially homogeneous and isotropic medium follow the equation of motion

$$\left(-\partial^2_{t_1} - m_\rho^2 - \mathbf{k}^2\right) D^{\mathrm{ret}}_{\rho,T}(\mathbf{k}, t_1, t_3) - \int_{t_3}^{t_1} d\bar{t}\Sigma^{\mathrm{ret}}_{\rho,T}(\mathbf{k}, t_1, \bar{t}) D^{\mathrm{ret}}_{\rho,T}(\mathbf{k}, \bar{t}, t_3) = \delta(t_1 - t_3). \tag{9}$$

The dynamic medium evolution is now introduced by hand via a specified time dependent retarded meson self energy $\Sigma^{\mathrm{ret}(\tau,\omega)}$ [6]. From that the self energy $\Sigma^<$, needed for solving equation (8), follows by introducing a background temperature of the fireball. The fireball is assumed to generate the time dependent self energy Σ^{ret} and, assuming a nearly quasi thermalized system, the ρ-meson current-current correlator $\Sigma^<$ is given via $\Sigma^<(\tau, \omega) = 2in_B(T(\tau)) Im \Sigma^{\mathrm{ret}}(\tau, \omega)$, which follows from the KMS relation [8], being valid for thermal systems. The latter is a rather strong assumption, but necessary in order to proceed.

III. THE MEDIUM IN NONEQUILIBRIUM

The medium effects are introduced via a specific evolving self energy of the vector meson. A simple self energy

$$\Sigma^{\mathrm{ret}}(\omega, \tau) = -i\omega\Gamma(\tau), \tag{10}$$

with a $\mathbf{k}$- and ω-independent width Γ, leads to a Breit-Wigner distribution for the spectral function. The time dependence is being accounted for by introduction of the parameter τ. For the $\mathbf{k} = 0$ mode, the full self energy for coupling to $J^P = \frac{1}{2}^-$ -resonances is given by

$$\Sigma_T(\omega, \mathbf{k} = 0) = \frac{\rho}{2} \left(\frac{f_{RN\rho}}{m_\rho}\right)^2 g_I \frac{\omega^2 \bar{E}}{(\omega + \frac{i}{2}\Gamma_R)^2 - \bar{E}^2} - i\omega\Gamma, \tag{11}$$

with $\bar{E} = \sqrt{m_R^2 + \mathbf{k}^2} - m_N$ and m_R and m_N the masses of the resonance and the nucleon respectively. Γ_R is the width of the resonance and g_I the isospin factor [9]. For our purpose it suffices to retain the part of the self energy that creates the pole structure. We will neglect the ω^2 in the numerator because it causes straightforward dispersion relations to become invalid (subtracted dispersion relations are needed in this case). We will shorten $\left(\frac{f_{RN\rho}}{m_\rho}\right)^2 g_I/2\,\omega^2$, with $\omega^2 = 1\mathrm{GeV}^2$ by C, a dimensionless factor.

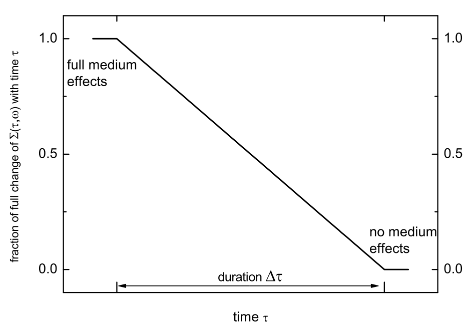

FIG. 2: Linear switching off of in-medium effects over a certain time $\Delta\tau$

Investigating which contributions in equation (5) come from which times in the past, shows that there are contributions from early times as well as alternating positive and negative contributions. An interpretation of this becomes difficult and it follows that only the time integrated yield is a physical quantity.

In order to quantify the times that the mesons' spectral properties need to adjust to the evolving medium we change the self energy linearly in time (see Fig. 2). As a possible characteristic timescale we consider the difference of the final spectral function to the dynamically calculated one at the time where the medium effects are fully turned off, described by the difference in the moment $\int_0^\infty A(\omega)^2\omega^2 d\omega$ of the two spectral functions or the difference in the peak position and height. All methods lead to similar results [6]. We find an exponentially decreasing difference with increasing duration of the change $\Delta\tau$ (see Fig. 2), from that we extract a time constant $\tilde{\tau}$. For the the ρ-meson we find a typical timescale of about 3 fm/c. That means that the spectral properties follow the changes in the medium nearly instantaneously only if the evolution is very slow as compared to 3 fm/c. This means that the vector mesons possess a certain memory of the past, and even if they decay outside the medium, they still carry information on the medium in that they were produced. This becomes important especially for the ω-meson, having a width of only 8.49 MeV ($\tilde{\tau} \approx 60$ fm/c). It turns out that $\tilde{\tau}$ is proportional to c/Γ_2 with c lying between 2 and 3.5, depending on m and Γ_1. This is significantly longer than the naively expected timescale of $1/\Gamma$. The time needed by the dilepton rate to follow changes is approximately equal to that of the spectral function. The quantum mechanical nature of the regarded systems leads to oscillations and negative values in the changing spectral functions, occupation numbers and production rates as well as interferences that one does not get in semi-classical, adiabatic calculations. The "rate" calculated here possesses the full quantum mechanical information incorporated and contains "memory" interferences that might cause cancellations - hence the rate has to be able to become negative while the time integrated yield always stays positive as the only observable physical quantity. An intriguing example for the occurring oscillations is shown in Fig. 3.

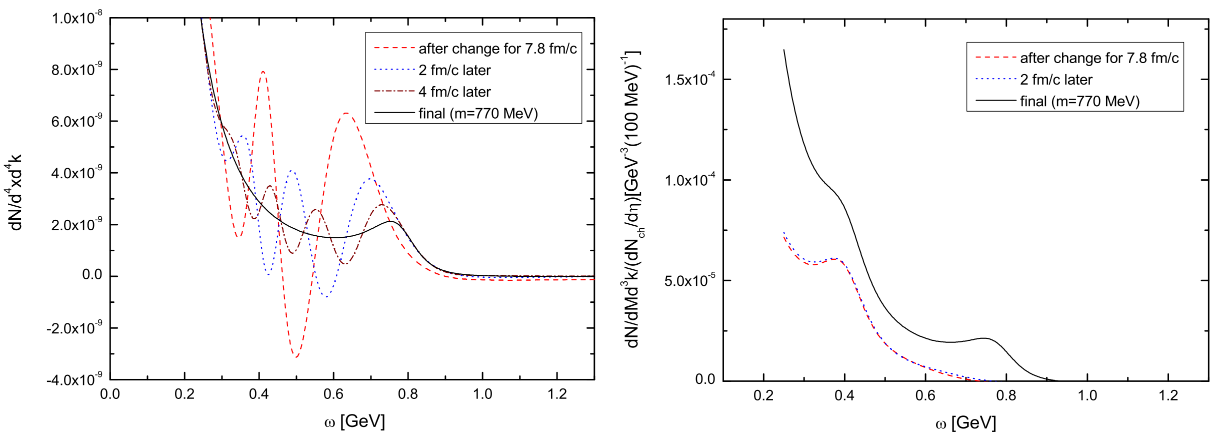

FIG. 3: Production rate for the change of the mass from $m = 400$ MeV to 770 MeV (constant Γ=150 MeV and constant $T = 160$ MeV) directly after the self energy has reached its final form (after 7.8 fm/c) and 2 (and 4) fm/c later. Oscillations and negative values appear in the intermediate rates (left). The corresponding yield stays positive (right).

To calculate the yield, we model the fireball evolution and fold it with the calculated time dependent rates, similar to [3]. For the effective volume we choose a longitudinal Bjorken expansion together with an accelerating radial flow

$$V_{eff}(t \geq t_0) = \pi ct(r_0 + v_0(t - t_0) + 0.5a_0(t - t_0)^2)^2, \tag{12}$$

with $r_0 = 6.5$ fm, $v_0 = 0.15$ c and $a_0 = 0.05$ c^2/fm (see also [10]). From (12) and the constraint of conserved entropy (given by a constant entropy per baryon $S/A = 30$ for SPS energies [11]), temperature $T(\tau)$ and chemical

potentials follow as functions of time. We start the calculation at the freezeout temperature of 175 MeV, whereas the final temperature, reached after a lifetime of about 7.8 fm/c is 120 MeV (thermal freezeout). At this point, we turn off further dilepton production by a rapid decrease of the temperature towards zero. With the time dependent temperatures we can integrate the rate and immediately find the yield per unit four momentum. The results for different scenarios are shown in Fig. 4. We compare to Markov calculations that assume instantaneous adaption of the spectral function (and the rate) to the self energy, as employed in [3] (dashed lines in Fig.4). The yield resulting from a mass shift following from assumed Brown-Rho scaling [12] shows an enhancement of about a factor of 2 in the dynamic calculation within the mass range where the CERES experiment [13, 14] has measured a strongly enhanced dilepton yield compared to calculations with vacuum spectral functions. Also the coupling to the N(1520) resonance with no broadening shows an enhanced production around the resonance peak and the ρ vacuum peak, but also a reduced yield in the region between the peaks. On the other hand, the difference to Markov calculations becomes smaller for large in-medium widths.

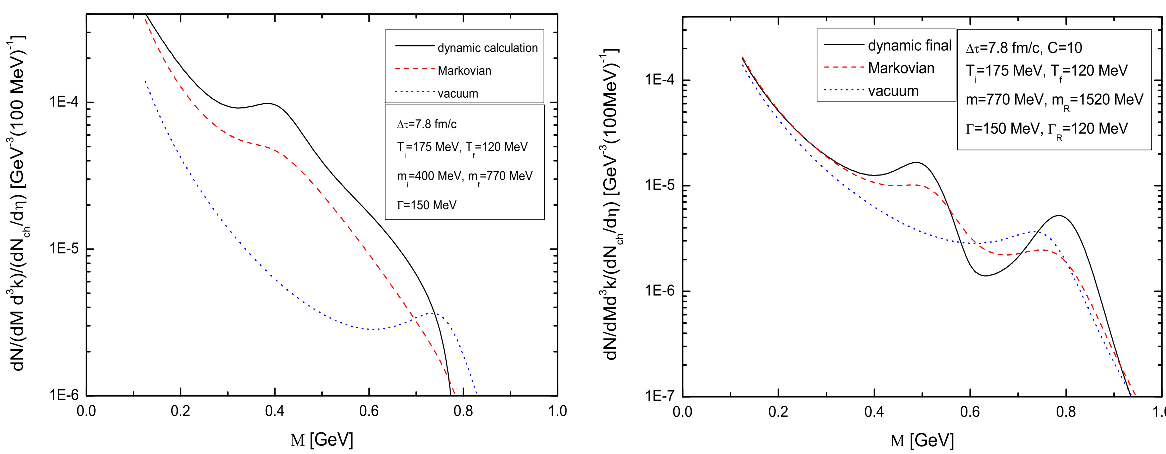

FIG. 4: Comparison of the dynamically computed dilepton yield from the fireball (solid line) to the one calculated assuming adiabaticty (as done in principle in [3])(dashed line), to show the differences caused by memory effects. Mass shift to 400 MeV in-medium mass (left) and coupling to the N(1520) resonance (right). No broadening of either the resonance or the ρ-meson is included here. Also shown is the yield coming from a constant ρ meson's vacuum spectral function (dotted line) for comparison.

IV. SUMMARY AND CONCLUSIONS

In the present work we introduced a method to calculate dilepton production rates within a non-equilibrium field theory formalism, based on the real time approach of Schwinger and Keldysh. We investigated possible medium modifications of the ρ meson in a fireball created in a heavy ion collision. We considered mass shifts, broadening and coupling to resonances. Special attention was put to possible retardation effects concerning the off-shell evolution.

The timescale on that the spectral function adjusts to changes in the self energy was found to be proportional to the inverse vacuum width of the meson Γ_2 like c/Γ_2, with c approximately 3. Further dependence on the in-medium width as well as on the size of the medium modification is present. We find typical retardations for the ρ of 3 fm/c and about 60 fm/c for the ω, a *very slow* adjustment.

The full quantum field theoretical treatment leads to oscillations in all mentioned quantities when changes in the self energy are performed. This oscillatory behavior reveals the quantum mechanical character of the many particle system, present in the investigated heavy ion reaction. The oscillations potentially cancel when the rate is integrated over time such that the measurable dilepton yield is always positive.

Comparison of dynamically calculated yields with those calculated assuming adiabaticity reveals differences. About a factor of 2 difference was found within the invariant mass range of 250 to 500 MeV for mass shifts predicted using Brown-Rho scaling for the ρ-meson in a fireball at SpS energies (158 AGeV). This is the range where CERES measures an increased dilepton yield as compared to calculations assuming the ρ's vacuum shape. Similar results were found for the coupling of the ρ-meson to resonance-hole pairs. Our findings show that exact treatment of medium modifications in principle requires the consideration of memory effects. Further investigation of the ω-meson is being done - due to its small width it causes numerical complications. It has to be seen how our results can be consistently (and probably in an approximative manner) incorporated in semi-classical nonequilibrium transport codes like UrQMD [15], BUU [1] or HSD [16].

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